Price formations by Monte-Carlo method: a simple comparison

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Abstract

This paper studies similarities and differences between models based on Monte-Carlo method by focusing on so-called “stylized facts”. In this study, we propose a model based on evolutionary algorithm and take the other model based on statistical physics. For this purpose, first we present a genetic learning model of investor sentiment and then several ordinary time series analyses are conducted after generating sample paths. Finally, the price properties are compared to those in the Ising spin model using the same algorithm. Our results show that both the Monte-Carlo simulations seem to lead to the similar dynamics reported in real markets in that the agents are boundedly rational or have some biases towards to the market. However, other time series properties are apparently different since the algorithm of price formation is different.

Keywords: Agent-based Computational Economics, Monte-Carlo Method, Stylized Facts, Biases

1. Introduction

Agent-based computational finance (hereafter ACF) has been an expected and feasible way to explain micro-macro relations in economic dynamics (Chen 2002; Tesfatsion and Judd 2006). The contributions of this area cover from tests of neo-classical economy to searching the possibilities to make more money. If we focus on some of the earlier studies which have shown time series properties, one can fall these studies into three groups in accordance with experimental setups and the methods of time series analyses; First, the artificial markets in which the external information is available for the agents can generate time series data at daily or longer time scale (LeBaron et al. 1999; Arifovic and Gençay 2000; Izumi and Ueda 2001). The algorithm in these models is usually evolutionary one. Second, in the models based on statistical physics, the agents can perceive mainly the information with respect to price/rate and therefore the data generated are usually considered as intraday ones (Hirabayashi et al. 1993; Sato and Takayasu 1998). We usually call these models “deterministic model”. Third, fundamentalist-chartist models, namely a fundamentalist thinks that the asset price converges to a fundamental price and a chartist follows patterns in past series, can generate both the daily and the high frequency data (Lux 1998; Lux and Marchesi 1999; Lux and Marchesi 2000), but this does not imply that the time series properties hinge on the number of fundamentalists (or chartists). Those previous studies, in addition, have not taken into account the empirical evidences from the questionnaire studies and the attitudes of speculators to market conditions or to their own unrealized profit/loss, as will be mentioned in the sequel.

On the other hand, the recent development of behavioural economics has enabled us to propose descriptive models with respect to the behaviours of speculators. Besides, some models are proposed from the experimental economic points of view (Izumi et al. 2005; Ueda et al. 2004), or by applying some evidences of behavioural finance for agent-based computational economic studies (Barberis et al. 2001; Levy et al. 2000). Accordingly, ACF needs to incorporate these findings and contributions into each of the models.

Therefore, the aims of ACF research are to offer the framework/paradigm of how the models are built, and then to contribute to the economic literature. In this study, before presenting such a framework, first we show the conditions requisite for genetic algorithm, which is often used as a learning method of agents in ACF models, to represent investor sentiment developed by Barberis et al. (A model of investor sentiment: hereafter MIS) (Barberis et al. 1998), and to explore the similarities and differences between the time series properties of estimated models and results in previous studies (Sznajd-Weron and Weron 2002). In other words, this paper tries to present a multiagent model based on evolutionary algorithm and to report what kind of models is proper to lead to dynamics observed in real markets using Monte-Carlo method and comparing time series properties.

The rest of this paper is organized as follows. The next section introduces a summary of MIS, and shows some conditions requisite for GA to describe MIS. Then, we explored the time series properties of generated sample paths using deducted variables of MIS in section 3, and compared time series properties to those of the Ising spin model in section 4. And finally, section 5 concludes this paper.

2. Description of Investor Sentiment by Genetic Algorithms

2.1. Background

Traditional economics postulate that an individual forms his/her subjective belief/probability using probability theory in face of uncertainty situations. At the same time, he/she is supposed to have perfect knowledge and an ability to deal with the whole information for decision makings. However, psychological and cognitive experiments reveal that people are bounded rational, not perfectly rational. That is, one's decision makings are often based on a small number of heuristic principles which makes it easier to assess subjective probabilities to figure and to make a prediction. While such a heuristic may provide people with originality and creativity, there is sometimes a cause of their decision bias.

Representativeness heuristic is one of the well-known cognitive biases (Tversky and Kahneman 1974). In neo-classical economy, people are supposed to behave rational in their decision makings by calculating objective probability from Bayes Theorem. On the other hand, representativeness heuristic is that people are likely to make their decision by bracketing a typical series among events in order to manage a huge amount of information quickly. In particular, he/she tries to find some patterns in random sequence. Several typical examples of this heuristic are base rate neglect, law of small number, gambler's fallacy, and hot hand fallacy.

Conservativeness, in contrast, is that people would not change his/her subjective belief soon when he/she knows a new piece of evidence (Edwards 1968). That is to say, while representativeness heuristic is observed when one neglects base rate of an event, conservativeness is seen when he/she overestimates it. This also implies that one does not use Bayes Theorem properly.

In fact, such phenomena are observed in real financial data, namely overreaction of investors corresponds to representativeness heuristic and under-reaction of investors is conservativeness (Barberis et al. 1998). The former event occurs when "the average return following not one but a series of announcements of good news is *lower* than the average return following a series of bad news announcement" (Barberis et al. p. 313). Barberis et al. illustrate several statistical evidences such as predictability of aggregate index returns or cross section of stock returns. On the other hand, the latter phenomenon means "that the average return on the company's stock in the period following an announcement of good news (i.e. after the initial price reaction to the news) is *higher* than the average return in the period following bad news" (Barberis et al. pp. 310-311). They introduce some empirical facts such as cross section of stock returns and stock price movements after the announcements and events.

2.2. A model of investor sentiment

Barberis et al. have developed their model of investor sentiment in order to describe representativeness and conservatism of people. The main assumptions are as follows:

(a) Return process follows a random walk.

(b) The investor in this economy does not realize that the process follows a random walk.

(c) He/She thinks that the economy is either in a stable state or in an unstable state. While the return process is supposed to be mean-reverting in the stable economy, the process is believed to have a trend in the unstable economy.

(d) The investor believes "an underlying regime-switching process" (Barberis et al. p. 319) and such a process rarely takes place.

More formally, the constitution of MIS is twofold; First, a market is either in a stable state or in an unstable one. If the market is in a stable condition, the probability π_H that the price movement will be the same as the previous one is over 0.5. While if the market is in an unstable state, the probability π_L is less than 0.5 (Table 1). The parameter λ_1 is the probability of transition from unstable condition to stable one, while λ_2 is the one from stable condition to unstable one (Table 2). Moreover, Barberis et al. postulate that the sum of λ_1 and λ_2 is less than unity and that λ_1 is smaller than λ_2. Second, the price movement in the economy is either +1 or -1. Therefore *q(t)*, the probability that the market is unstable, is renewed by equation (1) in case that the price movement is the different from the previous one, or by equation (2) otherwise (Figure 1).

Table 1: Transition probability

a. Unstable condition

	$\Delta S(t+1)=+1$	$\Delta S(t+1)=-1$
$\Delta S(t)=+1$	π_L	$1-\pi_L$
$\Delta S(t)=-1$	$1-\pi_L$	π_L

b. Stable condition

	$\Delta S(t+1)=+1$	$\Delta S(t+1)=-1$
$\Delta S(t)=+1$	π_H	$1-\pi_H$
$\Delta S(t)=-1$	$1-\pi_H$	π_H

Table 2: An abstract of investor's recognition

t＼t+1	Stable condition	Unstable condition
Stable condition	$1-\lambda_2$	λ_2
Unstable condition	λ_1	$1-\lambda_1$

$$q(t+1)=\frac{((1-\lambda_1)q(t)+\lambda_2(1-q(t)))(1-\pi_L)}{((1-\lambda_1)q(t)+\lambda_2(1-q(t)))(1-\pi_L)+(\lambda_1 q(t)+(1-\lambda_2)(1-q(t)))(1-\pi_H)} \quad (1)$$

$$q(t+1)=\frac{((1-\lambda_1)q(t)+\lambda_2(1-q(t)))\pi_L}{((1-\lambda_1)q(t)+\lambda_2(1-q(t)))\pi_L+(\lambda_1 q(t)+(1-\lambda_2)(1-q(t)))\pi_H} \quad (2)$$

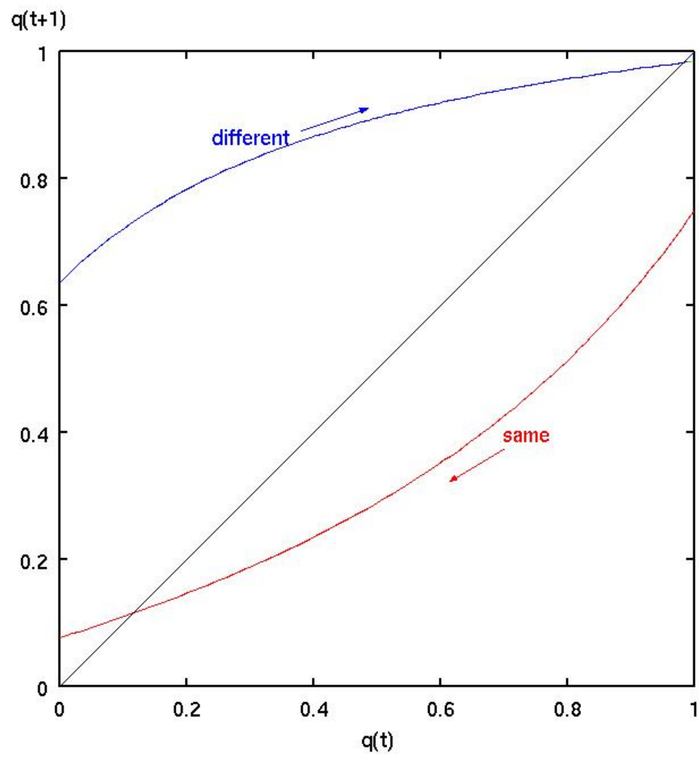


Figure 1: A model of investor sentiment

2.3. From genetic algorithm model to investor sentiment

GA has been often used as a learning method of agents in ACF models (Arifovic 2000; Arifovic and Gençay 2000; LeBaron 2000; Riechmann 2000). According to Arifovic, and Arifovic and Gençay, there are mainly four advantages of using GA; (1) it does not need highly computational abilities, (2) it can represent the heterogeneity of agents' belief, (3) whether a decision rule can survive or not depends on its performance, and (4) GA is able to mimic the behaviour of subjects observed in experimental economics (Arifovic 2000; Arifovic and Gençay 2000). However, the applications of GA to the ACF models are limited; (1) to select an optimal behaviour (Lawrenz and Westerhoff 2003; Marimon et al. 1990; Routledge 2001), (2) to react external information (LeBaron et al. 1999; Izumi and Ueda 2001), (3) to determine the time horizon (Bullard and Duffy 1998) and (4) to select future portfolio (Arifovic and Gençay 2000; Lettau 1997). It can, therefore, be said that the previous studies have not taken into account the attitudes of speculators to market conditions or to their own unrealized profit/loss as well as the other ACF models.

In this literature, we assume that an agent has five variables in order to represent the original MIS, and each of which is a binary bit with the following meanings:

(a) *judge*

If this variable is 1 then an agent considers the market to be stable. On the other hand, if that is 0, she does the market to be unstable.

(b) $stable_+$, $stable_-$, $unstable_+$, $unstable_-$

The variable $stable_+$ is used if the previous movement is +1 and an agent considers the market to be stable. If the variable is 1, she forecasts the next change is +1, while if 0 then -1. The same is true to other variables.

First, an agent judges the market condition by her judge, and then makes a prediction for the next movement, based on that value and on the previous change. Therefore, the roles of the agents in our model are to tell us the possibilities to represent a model of investor sentiment through their learning.

In order to check whether the proposed model describes MIS, we fed two kinds of price movement series, each of which was given iteratively, to the agents. The one is －＋－－－＋＋＋, and the other is ＋－. If one takes any three-period return series from the former set, one can find out that all of them are different from each other. He/She can thus consider this set to be the one given by the quasi-random numbers. That is to say, this set has at least the minimum information with respect to diversity. The latter series is given such that a price movement is always opposite to the previous and the next movement. Therefore, so long as GA works well, the probability q_t should be converged to a point illustrated in Figure 1.

We are now able to test whether the decision rule *judge* was equivalent to the theoretical $q(t)$ by feeding four kinds of price movements in the following; At first, we saw the movements of the *judge* by feeding the PMS 1 and 2. If the fluctuation of the *judge* was similar to that of the *q(t)*, then we thought that the GAL was able to represent the MIS.

The simulation was run by altering parameters of genetic algorithm, i.e. crossover (0.6 or 0.8), mutation (0.01 or 0.05), learning frequency (LF) (every period or every 19-period), time horizon, i.e. how long the sum of the fitness values is needed in selecting parents[1], and fitness calculation[2]. Hence, this simulation has totally 72 kinds of results and we obtained the following conditions; First, the agents needed to know a market condition for their learning. Second, the information used when the agents selected their parents must be up-to-date.

3. Price Formations

This section attempts to show the relations between the parameters of GA and time series properties by generating sample paths using estimated variables in the previous section and by applying them for several time series analyses.

3.1. Generation of sample paths

Since this simulation required only the variables of MIS, no agent existed or no parameter of genetic algorithm except learning frequencies was used.

The price movement $\Delta S(t)$ was determined using a coefficient α =2.0 by the equation below when the previous movement was +1;

$$\Delta S(t) = \begin{cases} \alpha(2p_u - 1) \\ -\alpha(2p_u - 1)/2 \end{cases}$$ (upper: if *rnd*() < p_u, lower: otherwise)

where $p_u = q(t)\cdot\pi_L + (1-q(t))\cdot\pi_H$ is the probability of price-up, and by the following equation when the previous change is -1;

[1] There are three types: (ha) The corresponding fitness value. (hb) Sum of the fitness values in the last 19 periods. (hc) Total fitness values.

[2] There are also three types: (fa) An agent receives +1 if she predicts the price change precisely. (fb) She receives +1 if she predicts the price movement and, at the same time, judges the market condition properly. (fc) She receives +1 if she judges only the market condition properly. While she receives +3 if her expectation is also right about the price movement.

$$\Delta S(t) = \begin{cases} \alpha(2p_d - 1)/2 \\ -\alpha(2p_d - 1) \end{cases}$$ (upper: if *rnd*() < $p_{d,}$ lower: otherwise)

where $p_d = q(t)\cdot(1-\pi_L) + (1-q(t))\cdot(1-\pi_H)$ and $rnd() \in (0,1)$ are the probability of price-up and uniform random number respectively.

Other setups are as follows:

(a) Number of sample paths: 100

(b) Total periods: 20000

(c) Initial *q(t)*: 0.5

(d) Variables of MIS: estimated ones in the previous section

(e) Renewal of *q(t)*: in accordance with equations in section 2.

(f) Renewal frequency of q(t): the same as learning frequency (section 2)

(g) Initial price: *S(0)=100*, *ΔS(t)=S(t)-S(t-1)*=1 or -1 (*t*=-1, 0) (*ΔS(t)*s at *t=-1, 0* were determined arbitrarily.)

3.2. Stylized Facts

Financial market data contain many statistical properties called “stylized facts” for which traditional economics is difficult to explain. Some of them are about price movements per se and others are the relations between trading volumes, and price movements or volatility. Since the dynamics in this literature is only about price fluctuations, we will focus on the following three properties which seems to be the most popular and significant facts and have been reproduced by some agent-based simulation models.

(a) Exchange rates and stock prices have almost unit roots.

A simple form of unit root property is that if a time series *Y* has unit root, the coefficient *b* of regressed AR(1), $y_t = a + by_{t-1} + \varepsilon_t$, is 1. In this case, time series *Y* is unstationary and has long memory. Moreover, the differential $\Delta y_t = y_t - y_{t-1}$ is stationary time series.

Unit root test is firstly developed by Dickey and Fuller (1979), and now several kinds of methods, for instance Augmented Dickey Fuller test (Said and Dickey1984) or Phillips-Perron test (1988), are proposed. But we omit the results of this item since all the time series seem to have unit root.

(b) Returns have fat-tailed distributions.

Return is defined as $r_t = \log x_t - \log x_{t-1}$, where x_t is asset price at time *t*.

Fat-tailed distribution is whose density function decreases in a power-law fashion, and this fact is seen for returns at weekly or shorter time scale (Lux and Marchesi 2000).

(c) Returns per se cannot be predicted, namely they have almost zero autocorrelations.

Autocorrelations is calculated as

$$ACF(k) = \frac{\sum_{t=k+1}^{N} (r_t - E[r])(r_{t-k} - E[r])}{\sum_{t=1}^{N} (r_t - E[r])},$$

where *N* is the length of sample data and *r* is a sample mean.

(d) Return distributions shows long memory, namely absolute or squared returns are significantly positive and decrease slowly as a function of the lags.

There are other methods to investigate the long memory characteristics. For instance, Hurst exponents is calculated by dividing the sample path into some parts and then by drawing out the data in each part of the mean value. If this value is over 0.5, then the time series has long memory. Otherwise, the time series is random walk (H=0.5) or mean-reverting (H<0.5).

3.3. Time Series Properties

This part reports the results using the five parameter sets of 16 estimated models (Table 3). To conduct analyses, 1-term return and 8-term return, were employed.

Table 3: Estimated models of investor sentiment

	λ_1	λ_2	π_L	π_H	Crossover	Mutation	LF	Fitness
(a)	4.14E−03	7.14E−03	0.451	0.510	0.8	0.01	1	Fb
(b)	4.52E−03	6.16E−03	0.468	0.517	0.6	0.01	1	Fb
(c)	4.38E−02	5.11E−02	0.438	0.511	0.8	0.05	1	Fb
(d)	4.03E−03	4.75E−03	0.462	0.523	0.8	0.01	19	Fb
(e)	5.09E−03	1.26E−02	0.406	0.509	0.8	0.01	1	Fc

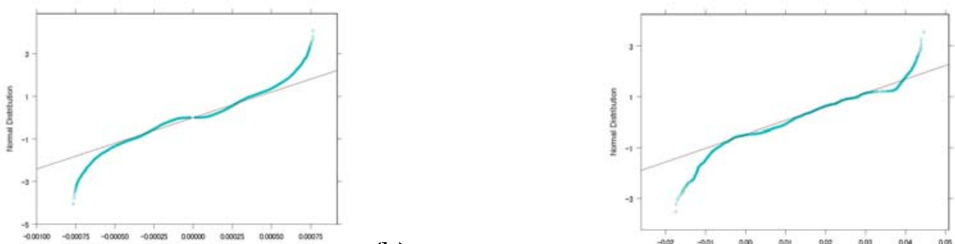
First, Figure 2 shows normal probability plots of both the return series. From these figures, the fat-tailed distribution is only observed when one takes 8-term return series. The fact that the parameters π_L and π_H are close to 0.5 may make the views of investors unchanged and thereby result in similar price movements. On the other hand, the fat-tailed distribution emerges for 8-term return series so long as investors have more chances to contact other market participants for their learning.

Next, Figure 3 illustrates the auto-correlation functions for return series and absolute return series. The first-order values of 1-term normal return series take significantly negative in most cases and long-term auto-correlations are found for absolute return series. These are relatively similar to the findings in econophysics (Sazuka et al. 2003). On the other hand, when it comes to 8-term return compared to Sznajd model (Sznajd-Weron and Weron 2002), both the auto-correlation functions do not have longer-memory except parameter (d). However, these values result from just the poor learning chances.

Then, Table 4 reports the BDS statistics which tests the null hypothesis of i.i.d. process. For both the two segments, the statistics rejects the null hypothesis except some cases, but for the 1-term return series, the values are much larger than those in Arifovic and Gençay. Combining the findings and the previous analyses, it seems that the time series properties of 1-term return series are similar not only to those of daily data but also to those of high-frequency data. On the other hand, the reason why the 8-term return series were not the same as those in Sznajd model is the way of price formation. The possible reasons could be considered; First, while the setups in our model are to determine the price movement, those in Sznajd-Weron and Weron are to form the price. Besides, it is because our model is not restricted by the condition that the prices created in the Ising spin model range just from 0 to 1.

Fourth, Table 5 presents the estimated values of Hurst exponents and Lyapunov ones respectively. The former method tests whether a time series has longer memory or not, and the exponent lies in one of the three aspects: persistent time series in case of $H > 0.5$, random walk in case of $H = 0.5$, and mean-reverting in case of $H < 0.5$. First, the estimated exponents of 1-term return series are insignificantly different from those of white noise, but less than those of 8-term return. This is because about half of market participants consider the market as unstable. In other words, since *q(t)* is around 0.5, the price movement is likely opposite to the previous one. Second, the Hurst exponent of 8-term return series is larger than 0.5 and similar to those in Sznajd model. Therefore, in this respect, both the models based on the Monte-Carlo method could generate or reproduce real market data. While if the GA-based model under investigation is dissipative, the positive Lyapunov exponent is a sufficient condition for the existence of chaotic dynamics, say if the estimated exponent is significantly positive then the time series is chaotic, while that results from noise in case that the exponent is less than 0. Unlike the results in Arifovic and Gençay (Arifovic and Gençay 2000), some values reported in this table show that time series data generated have not had such a dynamics. Especially, the series of 8-term residuals regressed by AR(10) except parameter set (d) reveal that they are not chaotic, while all the series of 1-term residuals are chaotic.

(a)



(b)

(c)

(d)

(e)

Figure 2: CDF plot of return series (left panel: 1-term return, right panel: 8-term return)

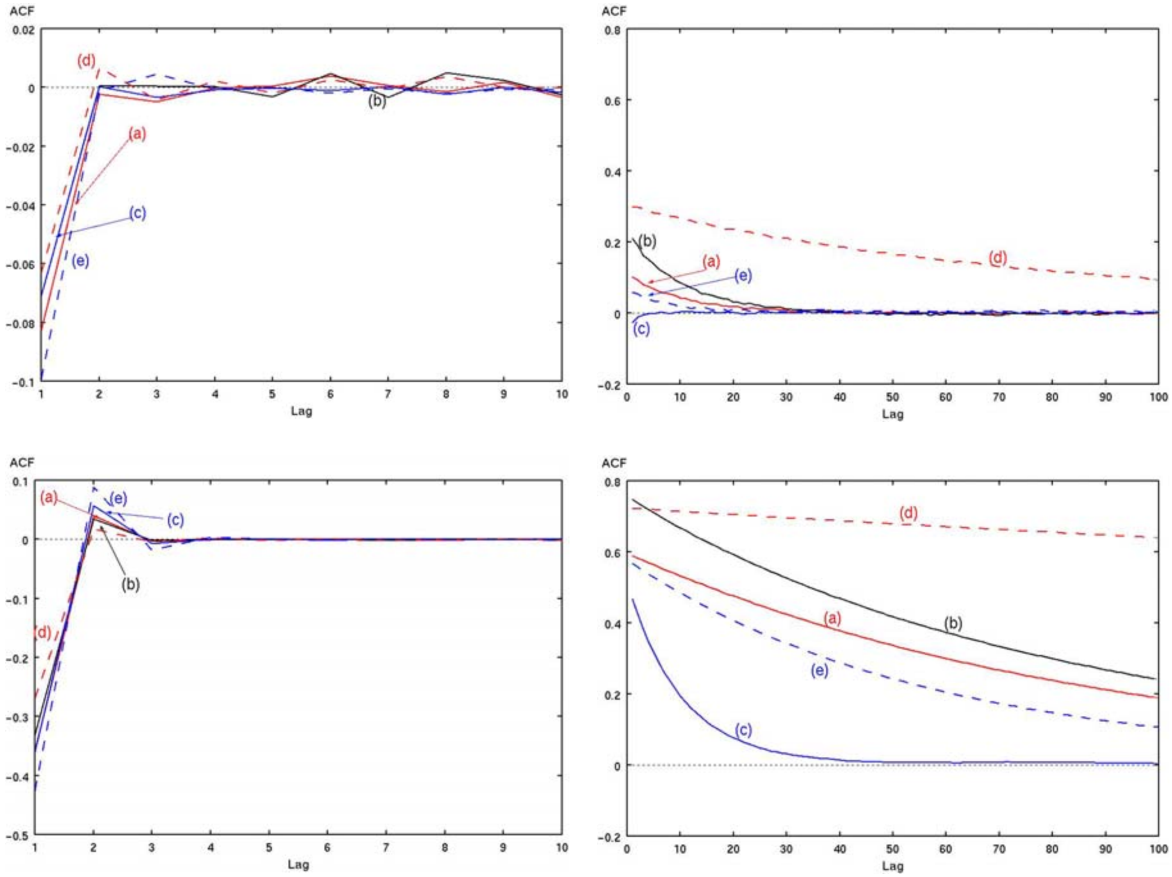


Figure 3: Auto-correlation functions (top left: 8-term return, top right: 8-term absolute return, bottom left: 1-term return, bottom right: 1-term absolute return)

Finally, consider the relations between parameters of GA and the statistical properties. First, the differences of crossover, mutation and learning frequencies seem to determine whether a return series has memory or not. Besides, a parameter set with higher mutation made the 8-term return series not rule out the lack of persistence. Second, actual market participants may adjust their opinions judging from the differences of learning frequencies. In other words, no matter how a volatility clustering might be seen, the generated sample paths with the parameter set (d) were not similar to actual data.

Table 4: BDS statistics

a. Return series

	1−term return				8−term return			
	ε =0.25σ		ε =0.75σ		ε =0.25σ		ε =0.75σ	
	m=2	m=3	m=2	m=3	m=2	m=3	m=2	m=3
(a)	698.00	1365.82	483.76	568.75	7.25	11.35	6.22	8.95
(b)	554.37	925.83	107.45	153.38	12.24	21.00	15.16	24.48
(c)	1158.62	1914.89	500.85	498.91	0.46	0.22	−1.22	−1.33
(d)	672.18	1721.63	254.18	377.62	24.52	49.63	18.66	27.89
(e)	1192.64	2237.21	679.47	782.35	5.26	7.45	4.39	6.17

σ is the standard deviation of the return series, and ε is the distance parameter.

b. AR(10) residuals

	1−term return				8−term return			
	ε =0.25σ		ε =0.75σ		ε =0.25σ		ε =0.75σ	
	m=2	m=3	m=2	m=3	m=2	m=3	m=2	m=3
(a)	2416.51	5782.78	732.03	1041.07	21.68	41.05	16.74	25.10
(b)	926.14	1637.08	548.36	630.91	4.98	8.17	6.10	8.84
(c)	227.35	368.34	2584.50	2530.67	−4.24	−3.66	−1.53	−1.60
(d)	1183.73	2639.77	694.90	979.20	25.49	51.15	19.42	28.90
(e)	1865.18	4100.71	1087.09	1575.60	9.73	12.91	10.16	13.07

Table 5: Estimates of Hurst exponents and Lyapunov exponents

	1−term return			8−term return		
	Hurst	Lyapunov		Hurst	Lyapunov	
	R/S	Return	AR(10)	R/S	Return	AR(10)
(a)	0.48	−1.04E−05	3.15E−05	0.52	−2.20E−04	−1.50E−04
(b)	0.47	−6.97E−06	4.48E−05	0.51	−6.68E−04	−1.80E−04
(c)	0.47	4.62E−07	4.15E−05	0.51	5.06E−06	−2.10E−04
(d)	0.48	2.06E−05	4.74E−05	0.52	7.09E−05	1.80E−04
(e)	0.47	1.86E−05	2.28E−05	0.51	1.73E−04	−1.10E−04

4. Discussion

4.1. Sznajd model

Sznajd model is originally one-dimensional and horizontal chain of Ising spin model where each spin stands for one of the two states, up-spin or down-spin. Usually, there are two spin evolution rules proposed. First, if a randomly selected pair has the same opinion, then the two neighbours share the same orientation. Second, if the pair has an opposite opinion, then each of the two neighbours "adopts their opinions from the second nearest neighbours." Therefore, a closed community consists of two contradicting/opposite opinions. The main applications of this model are voter model, small-world networks and so on (for more details, see Behera and Schweitzer, 2003).

In applying Sznajd model for financial markets, Sznajd-Weron and Weron have modified the second rule as follows: if the selected pair has the different opinion, the opinion of two neighbours takes one of the two directions randomly (Table 6). In this setup, the up-spin is considered as bullish and supposed to do buy-order. On the other hand, down-spin represents bearish and then do sell-order. If the market is more bullish (bearish), then the price goes up (down). Moreover, there is the third market participant in the economy, one fundamentalist. Though all the spins behave as trend-chaser, a fundamentalist knows the system and trade such that the price goes to a fundamental value.

The price is formed as follows: First, calculate the difference between up-spins and down-spins, say a market-clearing price is defined as the difference between demand and supply. In their literature, the price x_t is written as $x_t = \sum_{i=1}^{N} S_i(t)$, where N is the population size and $S_i(t)$ is the opinion of spin i. Second, the model determines whether the fundamentalist takes part in the economy, i.e. he/she will buy (take +1) with probability $|x_t|$ if x_t <0, and sell (take -1) with probability $|x_t|$ if x_t <0.

According to the presented paper, the dynamics generated by the above procedure have successfully replicated the characteristics of actual financial series, fat-tailed distribution of returns, long memory property, and unpredictability of normal returns. At the same time, our additional test proved unit root property and rejected the null hypothesis of i.i.d. process. Besides, these stylized facts are observed for not only 8-term return series but also 1-term ones.

Table 6: Opinion updating rule in Sznajd financial market model
('?' is a wild card.)

	T				->	t+1			
	i−1	*i*	*i*+1	*I*+2		*i*−1	*i*	*i*+1	*i*+2
(1)	?	+	+	?		+	+	+	+
(2)	?	−	−	?		−	−	−	−
(3)	?	+	−	?		?	+	−	?
(4)	?	−	+	?		?	−	+	?

4.2. Similarities and differences

Table 7 is a brief comparison between the two models and more similarities are considered as follows: (1) investors are not perfectly rational, say the agents decide their own behaviour by only the past price movement or the views of their neighbours, (2) the price is formed by the Monte-Carlo method, and (3) both the models do not assume any typical time scale. Though both the models are good approximations of actual financial time series, there are several differences in terms of model-setups. Especially these differences are classified into two aspects; The one is biases of agents and the other is the way the price determines.

As explained in the preceding section, a model of investor sentiment assumes that an individual is one of the two biases, representativeness heuristic and conservatism. In other words, while an overreacted investor is considered as a trend-chaser, an underreacted one is likely a contrarian. On the other hand, an agent in Sznajd model keeps bullish or bearish, say optimist or pessimist, so long as his/her spin shifts from one direction to another. Besides, changes in opinion are not relevant of changes in prices. Similar discussions are true for the item “reaction to prices.” Moreover, the dynamics in Sznajd model results in either “ferromagnetic” or “anti-ferromagnetic”, i.e. the homogeneity emerges in place of heterogeneity. Which may be that once bubble or anti-bubble situation emerges, the trend lasts and never dies. Whereas the proposed model can permit that the $q(t)$ departs from one of the two convergent points if each of the past price movements is always same as or different from the previous one. In other words, the operator “mutation” helps the genetic learning model have heterogeneity. Therefore, we can observe the everlasting price dynamics or a huge amount of time series, say high-frequency data. On the other hand, the Monte-Carlo based models of price formation, independent of whether a model on the statistical physics/econophysics, has an inevitable drawback with respect to the setups of agents. As pointed in Durlauf (Durlauf 2005), one need to take much care of whether his/her model can take into account heterogeneity of agents. Which means, while the implication of each spin does not always imply the actual behaviour of market participants, the fluctuation of $q(t)$ in the genetic learning model of investor sentiment, seems to have difficulties in describing the herd behaviour, for example, too.

Meanwhile the difference of price formation procedure seems to come from the possibilities of the effect of minorities or fundamentalist; The proposed model assumes that the stronger the market sentiment is, the more the price goes to the expected direction. Whereas the Ising spin model postulates that the broader the bid-ask spread is, the more the price is likely to go to another direction. Thus, that means there are two different presumptions. The one is whether a fundamental value exists or not in a market, and the other is how a fundamentalist or a minority has power to the market. While this simulation model puts more weight on the biases of market participants, the statistical physic model believes that the price converges to a unique fundamental value in the long run. In other words, that difference has the same meanings as “Are fundamentalists really rational?” (Hommes 2001)

Table 7: Comparison between models

a. Assumption and procedure

	GA and investor sentiment	Sznajd model
Behavior of agents	Trend follower or Contrarian	Optimist or Pessimist
Market power	Market sentiment	Fundamentalist effect
Price formation	Changes in price	Price per se
Updating or learning	Global	Local
Reaction to the market	Partially Yes	No

b. Dynamics

	GA and investor sentiment	Sznajd model
Unit root	Observed	Observed
Fat-tailed dist.	Partially observed	Observed
Long memory	Partially observed	(Partially) observed
IID process	Rejected	Rejected

4.3. Further discussion

Despite of some similarities and differences mentioned above, there are still several open questions remained. Some of them are for building the theoretical framework of artificial exchange market, and others are how to apply findings of behavioural finance for general social simulation models. Probably, taken into consideration that over five years have passed since LeBaron's expectation was publicized (LeBaron 2001), the spillover has been developing in one way or another.

One possible question of the first issue is whether the relation between time scale and learning in each model is valid (LeBaron 2000, 2006). Both the simulation models do not presume typical time scale, but resulted in that each time step was hourly long. However, there are not any good reason for that relation. On the contrary, the time scale in the GA simulation model, from hourly to monthly, is not consistent with the assumptions in the original model of investor sentiment, almost yearly. Likewise, it is not a good guess that only a few agents in Sznajd model have chances to update their opinion. In this respect, one needs to incorporate the empirical evidences and findings in terms of opinion formation/updating into the model setup.

5. Conclusion

This paper attempts to show whether genetic algorithm can represent a descriptive model of investor sentiment and what distinctions such an agent-based model has compared to the model based on statistical physics. For these purposes, we combined investor sentiment with genetic learning in an agent-based computational economic model, conducted time series analyses using sample paths generated, and compared them to those in the previous studies. Both the time series statistics reveal that a proper setup could lead to dynamics reported in the early studies no matter how the setups are different. Especially, investors are likely to react to a piece of new information and adjust their views to the market. On the other hand, the differences of price formation, even if the agents are boundedly rational, could result in different time series properties in some regards.

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[3] For more information, see http://www.absss.titech.ac.jp/.

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